Vector-Based Word Problems

How to Identify

A vector problem has two major components: direction and magnitude. Usually, the direction will be given by an angle between 0° and 360°, as if we are looking down at the problem from above. Sometimes, it will be given as a cardinal direction (positive x-axis for East, positive y-axis for north, etc). Magnitude will often take the form of object speed, wind speed, or water current speed.

Prerequisites

1) Basic Trigonometric Functions: Students should be comfortable setting up sine, cosine, and tangent identities from a given right triangle, as well as inverse trigonometric functions should the angle (direction) be unknown.
2) Vector Addition: Vectors are added component-wise. That is, for a vector $\mathbf{u} = \langle x_1, y_1 \rangle$ and another vector $\mathbf{v} = \langle x_2, y_2 \rangle$, $\mathbf{u}+\mathbf{v} = \langle x_1+x_2, y_1+y_2 \rangle$. Note—the angle of $\mathbf{u}+\mathbf{v}$ is not the sum of the two angles.
3) Pythagorean Theorem: Students should be familiar with $a^2 + b^2 = c^2$.

Process

1) The student should identify what pieces of information are given and what the question is asking for. **Key pieces:** vector 1, vector 2, and the resultant vector. Each vector will have magnitude, direction, and <x,y> coordinates. Writing out each piece can help the student focus on what is missing.
2) The student should draw out the vectors, labelling each piece of each vector. This will help the student visualize the problem, as well as give a general range of the answer.
3) If the student is struggling, redraw the triangle created by the resultant vector.
    a) If the component form is known, draw the triangle with the two values of its sides. Prompt the student with questions like, "If we know two sides of a right triangle, how do we compute the third?" to find magnitude. Prompt with questions like, "What would the tangent of this angle be?" and "How do we undo a tangent to get θ by itself?" to find the angle.
    b) If the magnitude and direction are known, draw the triangle with its hypotenuse and angle, and label the base "x" and the height "y". Prompt the student with questions like, "What would the sine/cosine of the angle be?" and "How do we get x/y by itself once we have set up the trig ratio?" to find the component form.
4) Make sure to tie the answer back to the context of the original problem. This will help reinforce the solution as having a real world meaning, rather than just being random math calculations.

Conclusion

The setup is the most important part of these problems. If the student does not understand the given information or mislabels a piece of a vector, the answer will be wrong. Calculation is not important in this type of problem, as it will be plugged into a calculator. The more the student practices, the easier this should become, to the point that it is almost formulaic. If the student is still struggling after repeated practice, the problem may lie with one of the prerequisite skills, and could use supplemental material in those skills before returning to these word problems.