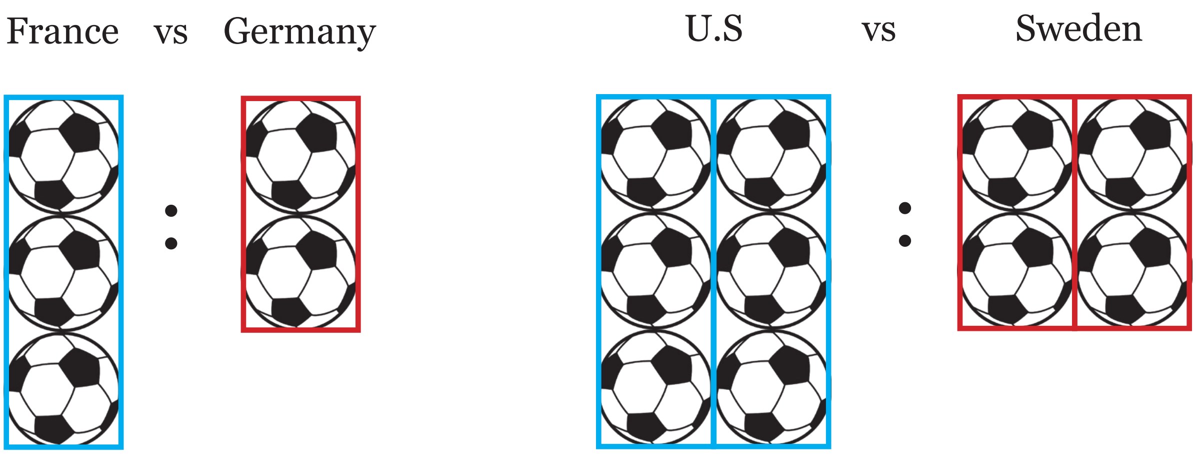
Defining a Ratio

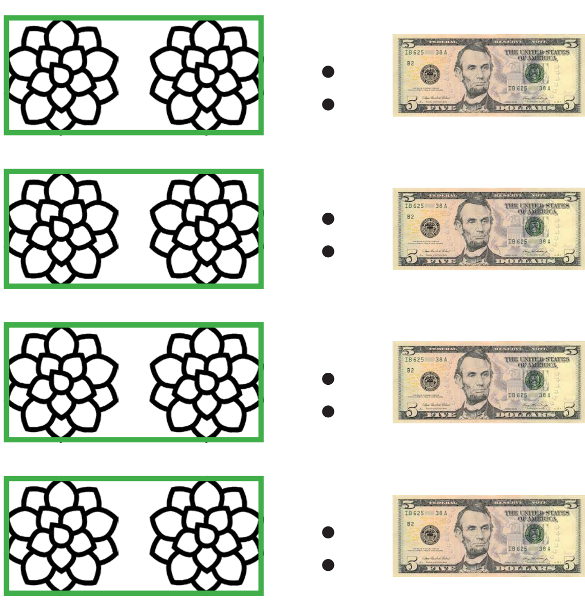
A ratio is just a comparison between two groups. Furthermore, two comparisons with different groups may have the same ratio. This happens when one comparison is a "multiple" of the other.

Example: In the 2019 World Cup, France beat Germany 3 points to 2 (don't fact check this). In the same tournament the U.S. beat Sweden 6 points to 4. Both France and the U.S. beat their opponent with a 3:2 (3 points to 2 points) ratio.



In both games, for every 3 points the winning team scored, the losing team scored 2 points. In the U.S. vs Sweden game, this happened twice. A 9:6 game, 12:8 game, and 45:30 game all have the same ratio. (You can check this by making groups of 3:2.)

Example: Lowes is having a sale. Succulents are 2 for \$5. You decide to buy some for your 8 closest friends and calculate it will cost \$20 in total. You just created a ratio.



That's right! Money works on ratios, too. If an item costs a certain price, that price remains constant no matter how many you buy. But even if you buy 148 succulents for \$340 (you *really* like succulents in this scenario), the ratio is still \$5 for every 2 succulents

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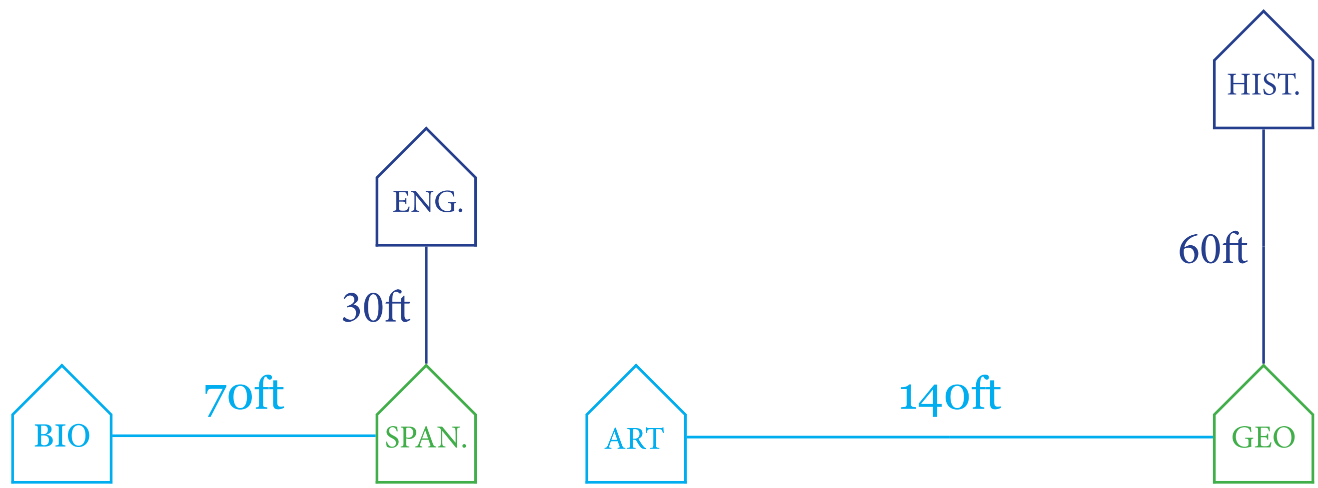
Extending the Definition

A ratio is not limited to groups. A ratio can be a comparison of two lengths. Your height compared to the height of your computer is a ratio. The length of your thumb compared to the length of your index finger is a ratio. The circumference of the Earth compared to the circumference of the moon is a ratio. And, just like groups, two different comparisons of lengths may in fact be the same ratio.

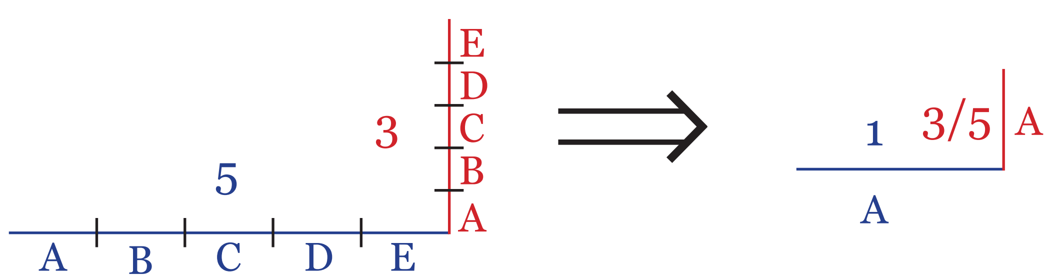
Example: In the morning, you walk 70ft from Bio to Spanish, then 30ft from Spanish to English. After lunch you walk 140ft from Digital Art to Geometry, then 60ft from Geometry to World History. Your walking paths have made a ratio.



Your first walk in the afternoon is twice your first walk in the morning, and your second walk in the afternoon is twice your second walk in the morning. Both ratios are the same: for every 7ft you walk on your first trip, you walk 3ft on your second trip. The distances could also have been 280ft:120ft, 35ft:15ft, or any other 7ft:3ft combination.

Note: When dealing with ratios, we can divide as easily as multiply. 70:30 breaks into 10 equal parts of 7:3. But what if we break it into equal parts of 7? 7 broken into 7 parts is just 1, and 3 broken into 7 parts is 3/7. Our new ratio is 1:3/7, OR the second trip is 3/7 the distance of the first trip. (If you don't believe me, start with 1ft:3/7ft and create 7 groups of each to get back to 7ft:3ft.)

Example:



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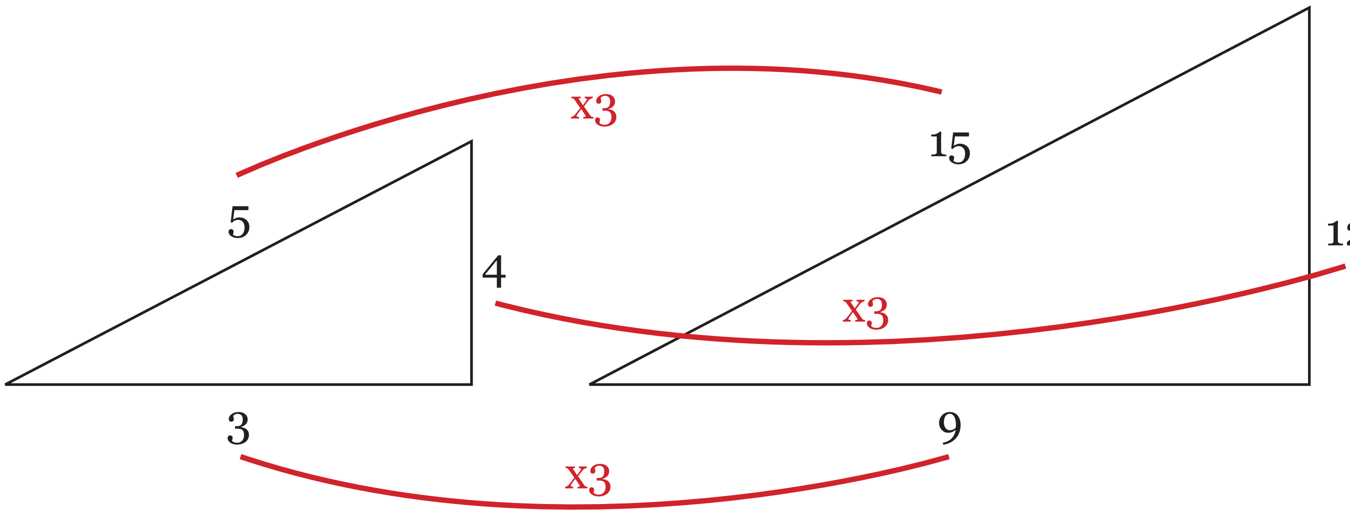
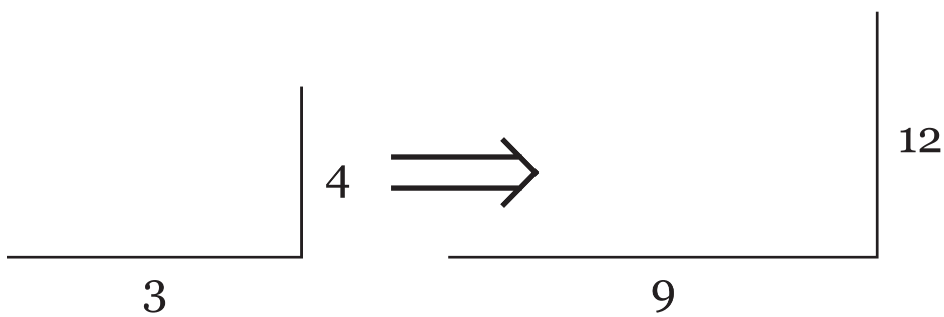
Similar Triangles

Corresponding sides of similar triangles also share proportions...



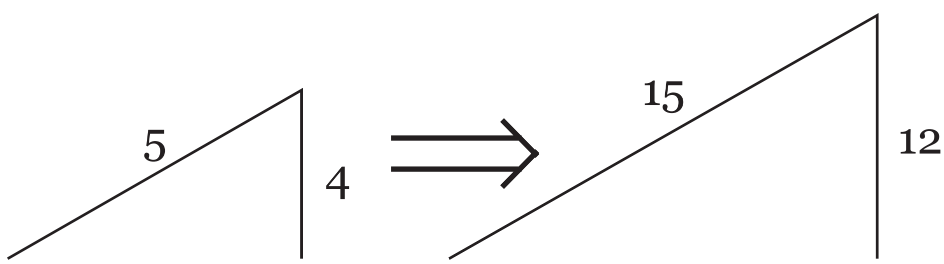
3:9 = 4:12 = 5:15 = 1:3

...as do pairs of corresponding sides.



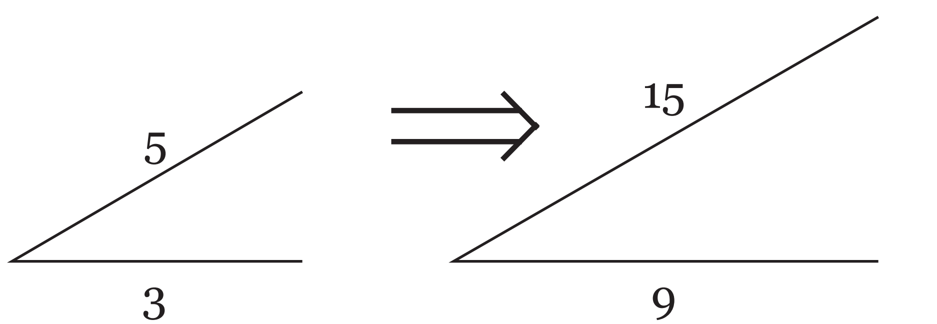
3:4 = 9:12

4:3 = 12:9



4:5 = 12:15

5:4 = 15:12



3:5 = 9:15

5:3 = 15:9

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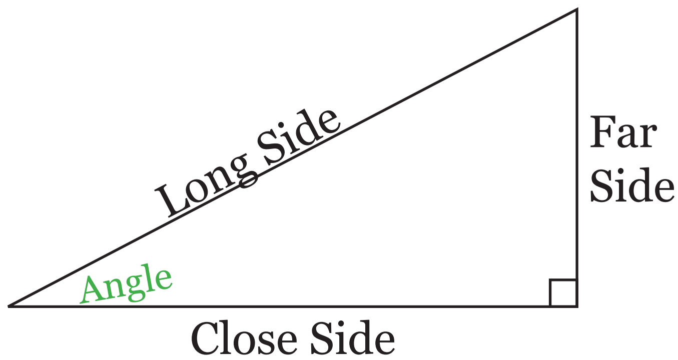
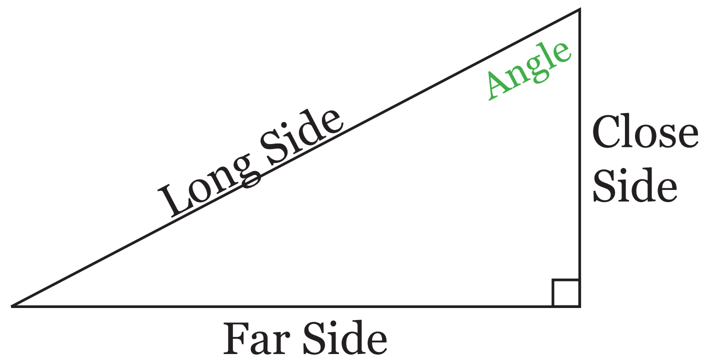
Right Triangles

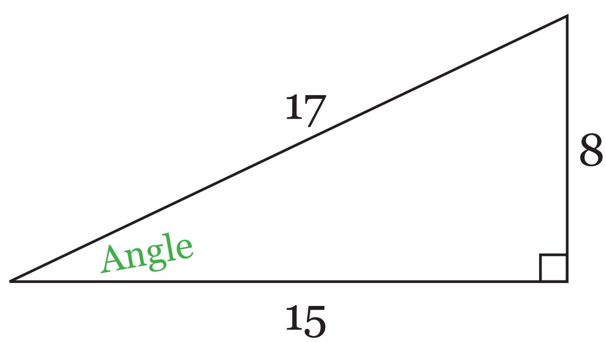
Given a right triangle and a non-right angle, we can assign a name to each of the sides. (Keep in mind that the names will change depending on which angle is selected.)





Notice how the long side remains the same, while the close side and the far side switch when we switch the angle chosen.

Example: Given a right triangle with sides 8, 15, & 17 and the angle farthest from 8, list out all possible ratios of sides.



Far Side : Long Side 8 : 17	Long Side : Far Side 17 : 8
Close Side : Long Side 15 : 17	Long Side : Close Side 17 : 15
Far Side : Close Side 8 : 15	Close Side : Far Side 15 : 8

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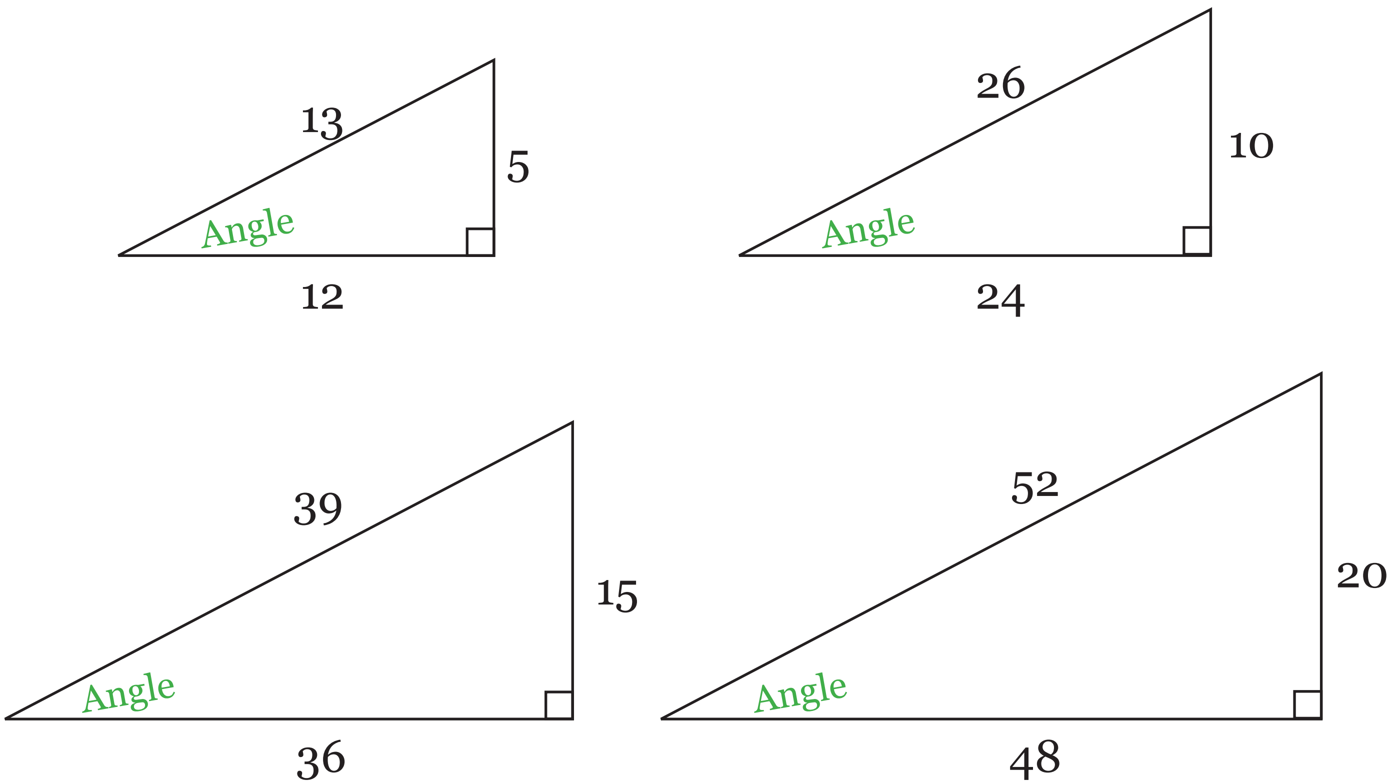
Combining Definitions

Now we can use the names of the sides to compare ratios of similar triangles.



Far Side : Long Side	Long Side : Far Side
5 : 13	13 : 5
Close Side : Long Side	Long Side : Close Side
12 : 13	13 : 12
Far Side : Close Side	Close Side : Far Side
5 : 12	12 : 5

Key Point: If two triangles share two angles, they are *guaranteed* similar (AA Similarity). The triangles above each share a right angle and one other angle (in green). Similar triangles share the same ratios (as above). **Therefore**, given a right triangle and non-right angle, that angle has a **unique** set of ratios that define that triangle and any scale factor of that triangle.

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Changing Terminology

Mathematicians are lazy, and they didn't want to write things like "The ratio of the far side to the long side of a given angle." As a result, they developed shorthand, a quicker way of writing the concepts we've been going over.

Opposite: The Far Side of the triangle

Adjacent: The Close Side of the triangle

Hypotenuse: The Long Side of the triangle

θ or α: The angle, when it's unknown (you may also see "x" or any other variable here).

Sine: Ratio of the Far Side (Opposite) to the Long Side (Hypotenuse) of an angle.

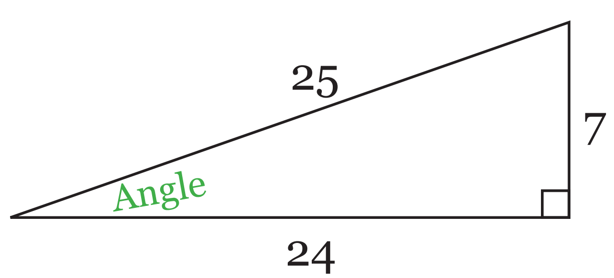
Cosine: Ratio of the Close Side (Adjacent) to the Long Side (Hypotenuse) of an angle.

Tangent: Ratio of the Far Side (Opposite) to the Close Side (Adjacent) of an angle.

Cosecant: Ratio of the Long Side (Hypotenuse) to the Far Side (Opposite) of an angle.

Secant: Ratio of the Long Side (Hypotenuse) to the Close Side (Adjacent) of an angle.

Cotangent: Ratio of the Close Side (Adajcent) to the Far Side (Opposite) of an angle.



Old Way: The ratio of the Close Side to the Long Side of the angle is 24:25.

New Way: Cosine(θ)=24/25

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Normal Words to Math Words

The ratio of the Far Side to the Long Side of an angle is Oppositve over Hypotenuse

sin(θ) = O/H

The ratio of the Close Side to the Long Side of an angle is Adjacent over Hypotenuse

cos(θ) = A/H

The ratio of the Far Side to the Close Side of an angle is Oppositve over Hypotenuse

tan(θ) = O/A

The ratio of the Close Side to the Far Side of an angle is Oppositve over Hypotenuse

csc(θ) = H/O

The ratio of the Long Side to the Close Side of an angle is Oppositve over Hypotenuse

sec(θ) = H/A

The ratio of the Long Side to the Far Side of an angle is Oppositve over Hypotenuse

cot(θ) = A/O

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